Econometrics Journal (2019), volume **20**, pp. 1–25.
Article No. ectj??????

Roy-model bounds on the wage effects of the Great Migration

John R. Gardner[†]

[†]*P.O. Box 1848, Department of Economics, University of Mississippi University MS, 38677, USA.*
E-mail: `jrgardne@olemiss.edu`
Received:

Summary This paper combines a Roy model of migration and counterfactual wages with racial differences in migration rates during the Great Migration to recover lower bounds on black-white differences in the wage impacts of northward migration. Identification is predicated on the idea that, when migration is more selective for whites, regional wage differentials for whites will be more contaminated with selection bias. In this case, the black-white difference in North-South wage differentials bounds the racial difference in wage impacts from below. Furthermore, as long as the impact of migration on whites' wages is nonnegative, a lower bound on the black-white difference in wage impacts is also a lower bound on the impact itself for blacks. Applying the identification result, I find that northward migration increased blacks' wages by at least 36% more than for whites', and hence by at least 36%, on average between 1940 and 1970.

Keywords: *Treatment effects, causal inference, differences in differences, Roy model, partial identification, bounds, Great Migration.*

1. INTRODUCTION

The African American Great Migration refers to a period spanning roughly 1915-1970 during which millions of southern-born blacks left their erstwhile homes in the southern US in favour of cities in the North. It is difficult to overstate either the size or significance of this event. Tolnay (2003), who provides a comprehensive review of the causes and consequences of the Great Migration, reports that over 2.5 million southern-born blacks lived in the North by 1970. In a seminal study on black relative economic progress, Smith and Welch (1989) find that migration from rural areas in the South to urban centres in the North played a pivotal role in black-white relative wage gains during the second half of the 20th century, explaining about as much of those gains as increased educational attainment among blacks. Collins and Wanamaker (2014) estimate that interregional migration explains roughly all of the relative wage gain for blacks between 1910 and 1930.

While there is widespread agreement that the Great Migration explains a significant fraction of the relative economic progress made by blacks during the 20th century, the extent to which observed regional wage differentials represent the causal effects of migration on wages has been a question of long-standing interest. The sheer sizes of the wage differences raise the question of whether they are partially a consequence of selective migration. Donohue and Heckman (1991), for example, report that in 1960 the black-white wage gap was 32% smaller for men living in the North than for those in the South. As Smith and Welch (1989) write:

> Even among men who have the same amount of education and job experience,

> large geographic wage differentials prevail among regions. Identifying their underlying causes is a complex empirical problem. Some of these wage disparities reflect cost-of-living differences between regions, or compensating payments for the relative attractiveness or undesirability of locational attributes (e.g. climate, crime, and density). Given the magnitude of the regional wage differentials we estimate, it is also likely that they proxy for unobserved indices of skill. Finally, the large black-white gap in the South may well reflect the historically more intense racial discrimination there.

adding that:

> If they proxy for unobserved skill differences, cross-sectional wage differentials would not represent the wage gain an individual would receive by moving from the South to the North.

In large part, what makes the question of the causal effect of northward migration on blacks' wages and relative wages difficult to answer is data limitations. Plausibly exogenous sources of variation in migration are difficult to come by and even more difficult to connect to data on wages. Few data sources contain information on migrants both before and after leaving the South, and few of the covariates present in available data (such as education, occupation, and industry) are exogenous to the migration decision, and not themselves potentially outcomes of migration.

Some of the best evidence that we have on the causal effect of the Great Migration on blacks' wages comes from a painstaking effort by Collins and Wanamaker (2014) to link black individuals' 1910 and 1930 Census records, allowing them to estimate first-differenced models of the wage effects of northward migration that allow for individual heterogeneity in wages and the migration decision. They conclude that there is little evidence of selective migration and that migration increased black migrants' wages by 63 log points during the period that they study, providing some of the first rigorous evidence on the causal effect of the Great Migration on wages. Illustrating the difficulties of analysing Great-Migration-era data, however, because the Census did not collect information on wages prior to 1940, the estimates in Collins and Wanamaker (2014) rely on imputed wage measures that relate occupational characteristics in 1910 and 1930 to occupation-specific wages in later Census periods. In addition, their estimates are limited to a subsample of individuals for which Census records for different decades can be reliably matched, and do not include estimates for white migrants.

In this paper, I take a different approach to inference about the causal effects of northward migration on black-white relative wages. While it was the African American Great Migration that was a powerful social force, it often goes unnoticed that many southern whites moved North during this period as well, albeit at substantially lower rates (I estimate below that on average between 1940 and 1970, there was about a 23% chance that a southern-born black man was living in North and about a 12% chance that a southern-born white man was). The results that I present below use this racial variation in migration rates to bound the selection bias present in comparisons between migrant and non-migrant southern-born blacks' wages.

This approach is motivated by a Roy model of migration, in which individuals migrate if the net benefit of doing so (which may reflect wages and regional amenities as well as psychological and social factors such as discrimination and family ties which affect the benefits and costs of migrating) is positive, and the counterfactual wages that individuals would earn in either the South or the North are positively correlated with this net benefit.

For example, suppose that wages in both the South and North are positively correlated with some unobserved skill, as is the net benefit from migrating (e.g., because the return to skill is higher in the North, or because skill reduces the cost of migrating). In this case, wage comparisons between migrants and non-migrants represent a combination of the causal effect of migration on wages and a bias term arising because migrants are a select group who would have commanded higher wages even if they had remained in the South.

Within this framework, a lower migration rate for whites implies that the migration decision is more selective for whites, suggesting that the selection bias component of the North-South wage difference will be larger for whites than for blacks. In this case, subtracting the North-South wage difference for whites from that same difference for blacks will over-control for the selection bias component of the black difference, recovering a lower bound on the black-white difference in causal effects of migration on wages. The theoretical results in this paper elucidate the conditions under which this intuition is correct.

Although it is not the object of identification in most studies of the Great Migration, a lower bound on the black-white difference in the causal effect of northward migration is informative for several reasons. It provides a sense of the causal impact of the Great Migration on racial wage convergence (rather than a combination of the causal effect and selection effects).[1] As Manski (1989; 1990; 2005) has argued, partial identification approaches allow us to conduct credible inference when the conditions required for standard econometric methods do not hold. In the Great Migration context, the critical concern is that wage comparisons between migrants and non-migrants overstate the causal effects of migration on wages and black-white relative wages. A conservative lower bound is sufficient to assuage this concern while still conveying the magnitude of the wage effects of northward migration. Finally, insofar as the causal effect of migration on wages was nonnegative for whites, a lower bound on the black-white difference in wage effects is also a lower bound on the wage effect itself for blacks.[2] The results that I present below suggest that, however migration affected wages for whites, it increased blacks' wages by an additional 36%, on average between 1940 and 1970, implying that northward migration increased wages for blacks by at least this amount during the period that I study.

The theoretical results in this paper relate to a large literature on sample selection problems, beginning with Roy's (1951) 'Some Thoughts on the Distribution of Earnings,' and including important papers on the econometrics of sample-selection, truncation and regime switching (see Tobin, 1958; Heckman, 1976, 1979; Amemiya, 1984). There is also a large literature on the identification of the Roy model itself and of treatment effects (and related objects such as the distribution of potential outcomes) from within a Roy-model framework (Heckman and Honoré, 1990; Eisenhauer et al., 2015; D'Haultfoeuille and Maurel, 2013b,a).

I develop the basic lower-bound identification argument in Section 2. In Section 3, I apply the argument to interpret racial differences in regional wage differentials as lower

[1] In addition, differencing the causal effect will remove the components of that effect that reflect regional price differences (as Smith and Welch, 1989 caution they might), to the extent that those differences are similar for blacks and whites.

[2] The sizes of the North-South wage differentials for whites (which are on the order of 30%, as I show below) coupled with what we know about the relatively industrial North suggests that the observed differentials cannot be explained by selection alone.

bounds on the causal effects of migration on wages, and compare those bounds to the results of standard point estimators. I conclude in Section 4.

2. MODEL

Let $r \in \{b, w\}$ index race, $l \in \{s, n\}$ index the choice of residential location in the South or the North, and x denote a vector of observed covariates that are realised before the migration decision is made (or otherwise unaffected by that decision).[3] Let y_l denote the potentially counterfactual wage that an individual would receive in location l, so that the observed wage can be expressed as $y = y_s 1(l = s) + y_n 1(l = n)$, where $1(\cdot)$ is the indicator function.

Let v denote the net utility benefit accruing to a southern-born individual from migrating to the North, and suppose that individuals migrate if $v \geq 0$ and remain in the South otherwise. The benefit v should be interpreted as reflecting the net benefit to migrating arising from all sources. For example, this benefit might include the wage gain $y_n - y_s$ (or the expected utility of this gain), non-pecuniary benefits to migrating (including regional amenities as well as, for blacks, the non-wage value of reduced discrimination in the North), the financial cost of migrating (which may depend on an individual's skill or ambition as well as both their origin and prospective destination), the psychological costs associated with leaving behind friends and family, and so on.[4] The identification argument developed below uses racial variation in the probability of migrating to bound the causal effect of migration on wages. The probability of migrating depends on the distributions governing the benefits v from migrating, which may be heterogeneous with respect to race and, potentially, a set of observed covariates. To model this potential heterogeneity tractably, I make the following assumption.

Assumption 2.1. *The race- and covariate-specific distributions of v belong to the same location-scale family, indexed by location parameters μ_{rx} and scale parameters σ_{rx}, for $r \in \{b, w\}$ and $x \in supp(X)$.*

Assumption 2.1 implies that the race- and covariate-specific benefits can be expressed in terms of a common random variable ε, drawn from the standard member of the location-scale family (that is, the member with location parameter 0 and scale parameter 1), as

$$v_{rx} = \mu_{rx} + \sigma_{rx}\varepsilon.$$

Consequently, a member of race r with covariates x will migrate if $\varepsilon \geq -\mu_{rx}/\sigma_{rx}$ and remain in the South otherwise. To connect this decision rule with observed racial differences in migration rates, note that a higher migration rate among blacks than whites

[3] Although it is common in the literature on the Great Migration to model the migration decision as a binary choice between remaining in "the South" or migrating to "the North," in reality the decision problem facing southerners was more complex. Because those originating from any location (state or city, say) in the South could potentially migrate to any other location in the South or North, in principle one could identify the wage effect for each origin-destination pair. Knowledge of these pair-specific effects would allow one to examine origin-destination-pair heterogeneity in the effects of migration, possibly helping to identify the underlying mechanisms through which migration increased (or failed to increase) wages. However, origin-destination-specific effects would be difficult to interpret without aggregating them to form an average wage effect (which is what methods that treat the migration choice as a binary decision identify).

[4] In this sense, the model can be viewed as a generalised or extended Roy model, such as those studied in Heckman et al. (2006) and D'Haultfoeuille and Maurel (2013b).

with covariates x implies that

$$-\frac{\mu_{bx}}{\sigma_{bx}} < -\frac{\mu_{wx}}{\sigma_{wx}}.$$

As Smith and Welch (1989) note, the sizes of the North-South wage differentials for both blacks and whites raise the concern that the decision to migrate is positively correlated with factors such as unobserved skill that help determine wages, in which case observed wage differentials may overstate the causal effects of northward migration on wages. Assumption 2.2, below, provides a simple way of capturing this potential dependence.

Assumption 2.2.

$$E(y_l|r, x, \varepsilon) = \alpha_{lrx} + \beta_{lrx}\varepsilon,$$

with $\beta_{lrx} \geq 0$ *for each* $l \in \{s, n\}$, $r \in \{b, w\}$, *and* $x \in Supp(X)$.

Assumption 2.2 is a natural way of modelling concern about positive selection into migration. It requires that the counterfactual wages an individual would earn in either the South or the North are (linearly) increasing in the net benefit that would accrue to that individual were they to migrate, whatever the source of that benefit may be. For example, wages in both locations may depend on unobserved skills such as cognitive ability, physical strength, or work ethic. If the returns to these skills are greater in the North, or if these skills make migration less costly, those who stand to gain the most by migrating North would earn relatively high wages in either location.[5] At the same time, Assumption 2.2 allows for the possibility that the relationship between counterfactual wages and the benefits from migrating differ by race (and by covariates), for example if blacks of all skill levels are attracted to the relatively tolerant North, attenuating the relationship between migration and unobserved skill.

Similar assumptions have been made elsewhere in the literature on sample selection problems (see, e.g., Wooldridge, 2002, Assumption 17.1). In fact, when ε, y_s and y_n are drawn from a multivariate normal distribution, the linearity of Assumption 2.2 is automatic (and the model reduces to a switching or Type-5 Tobit model; see Amemiya, 1984). More generally, Assumption 2.2 can be viewed as a first-order approximation to a potentially non-linear conditional expectation function.

2.1. Identification

Under the present model, the observed difference between migrants and non-migrants of race r and covariates x can be expressed as

$$\begin{aligned} E(y|l = n, r, x) - E(y|l = s, r, x) &= E(y_n|n, r, x) - E(y_s|s, r, x) \\ &= E(y_n - y_s|n, r, x) + [E(y_s|n, r, x) \\ &\quad - E(y_s|s, r, x)], \end{aligned} \tag{2.1}$$

[5] One limitation of this model is that it remains tacit about the underlying determinants of counterfactual wages and the migration decision, other than allowing that they may be related. One one hand, this makes it difficult to use the structure of the migration decision to make inferences about the causal effects of migration on wages. On the other hand, it allows migration to be modelled as a utility-maximising decision without placing strong restrictions on the migration decision itself (such as the utility function that the migration decision maximises, the arguments to that function, etc.).

where I have used "s" and "n" as shorthand notation for conditioning on the events "$l = s$" and "$l = n$," respectively. The first term in (2.1) is the average effect of the treatment on the treated, or ATT. This is the causal effect of interest; it represents the average difference between northern and southern counterfactual wages for those who choose to migrate. The second term in (2.1) is the source of concern that wage comparisons between migrants and non-migrants might be contaminated by selective migration. It represents selection bias arising because the counterfactual wages that migrants would have earned had they remained in the South might be systematically different than the observed wages of those who did, in fact, remain in the South.

Under Assumptions 2.1 and 2.2, (2.1) can also be expressed as

$$\begin{aligned}
E(y|n,r,x) - E(y|s,r,x) &= E\left(y_n - y_s|\varepsilon \geq -\frac{\mu_{rx}}{\sigma_{rx}}, r, x\right) + \left[E\left(y_s|\varepsilon \geq -\frac{\mu_{rx}}{\sigma_{rx}}, r, x\right)\right. \\
&\qquad \left. - E\left(y_s|\varepsilon < -\frac{\mu_{rx}}{\sigma_{rx}}, r, x\right)\right] \\
&= \underbrace{\left[(\alpha_{nrx} - \alpha_{srx}) + (\beta_{nrx} - \beta_{srx})\, E\left(\varepsilon|\varepsilon \geq -\frac{\mu_{rx}}{\sigma_{rx}}\right)\right]}_{\mathrm{ATT}_{rx}} \\
&\qquad + \underbrace{\beta_{srx}\left[E\left(\varepsilon|\varepsilon \geq -\frac{\mu_{rx}}{\sigma_{rx}}\right) - E\left(\varepsilon|\varepsilon < -\frac{\mu_{rx}}{\sigma_{rx}}\right)\right]}_{\mathrm{Bias}_{rx}}. \qquad (2.2)
\end{aligned}$$

As in (2.1), the first bracketed term in (2.2) represents the ATT while the second represents selection bias. Equation (2.2), however, uses the structure of the Roy model of migration to clarify the source of the selection bias: since $\beta_{srx} \geq 0$ by assumption, migrants, for whom the net benefit ε of migrating is relatively large, would earn higher wages even if they remained in the South.

Research designs based on conditional independence (alternatively, unconfoundedness or ignorability; see Rubin, 1974) assume that counterfactual outcomes (y_s and y_n here) are independent of treatment assignment (the migration decision here) conditional on a set of observed covariates. Were this the case in the present context, the location-, race- and covariate-specific intercepts α_{lrx} would absorb the correlation between counterfactual wages and the net benefit of migrating, the slope coefficients β_{lrx} and hence bias terms would vanish, and wage comparisons between migrants and non-migrants would identify the race$\times$covariate-specific wage effects of northward migration. However, the large regional wage differentials documented here and elsewhere persist even after controlling for observed measures of labor-market skill, raising the spectre of selective migration, even conditional on a rich set of covariates.

While both migration rates and observed North-South wage differentials were large for whites and blacks alike during the Great Migration period, the fact that blacks migrated at substantially larger rates than whites suggests a strategy for bounding bias arising because of selective migration. The intuition behind this approach is that if migration was more selective among whites (in the sense that fewer whites migrated), then we might expect the bias due to selective migration to be larger for whites as well. If this is the case, then subtracting the North-South wage differential for whites from that same differential from blacks, in effect, over-controls for selection bias among blacks. In this case, the black-white difference in North-South wage differentials represents a lower bound on the black-white difference in the average effect of migration on migrants' wages (the ATT). Moreover if, as the sizes of the observed regional wage differentials for whites and what we know about the relatively industrialised North suggests, the ATT is nonnegative for whites, the black-white difference in ATTs also represents a lower bound on the ATT itself for blacks.

To formalise this intuition, consider the black-white difference in North-South wage differentials which, following the above, can be decomposed into differential ATT and bias terms according to

$$\begin{aligned}[E(y|n,b,x) - E(y|s,b,x)] - [E(y|n,w,x) - E(y|s,w,x)]& \\ = (\mathrm{ATT}_{bx} - \mathrm{ATT}_{wx}) + (\mathrm{Bias}_{bx} - \mathrm{Bias}_{wx}).& \quad (2.3)\end{aligned}$$

From (2.2), the differential bias term in (2.3) will be negative (and the racial difference in regional wage differentials will bound the racial difference in ATTs from below) if

$$\frac{\beta_{sbx}}{\beta_{swx}} \leq \frac{E\left(\varepsilon|\varepsilon \geq -\mu_{wx}/\sigma_{wx}\right) - E\left(\varepsilon|\varepsilon < -\mu_{wx}/\sigma_{wx}\right)}{E\left(\varepsilon|\varepsilon \geq -\mu_{bx}/\sigma_{bx}\right) - E\left(\varepsilon|\varepsilon < -\mu_{bx}/\sigma_{bx}\right)}. \quad (2.4)$$

An additional assumption simplifies the analysis below.

Assumption 2.3. $\beta_{sbx} \leq \beta_{swx}$.

What Assumption 2.3 requires is that counterfactual wages in the South and the net benefit of migrating are less correlated for blacks than for whites.[6] Insofar as both blacks' and whites' migration decisions were made at least in part with respect to potential wage gains, this assumption is consistent with observed racial differences in migration rates. It is also consistent with what we know about economic discrimination against blacks in the South and about how racial wage gaps vary with observed measures of skill such as education during the Great Migration period.[7]

Because, under Assumption 2.1, a higher migration rate among blacks with covariates x than observably similar whites implies that $-\mu_{bx}/\sigma_{bx} < -\mu_{wx}/\sigma_{wx}$, Assumption 2.3 implies that the right-hand side of inequality (2.4) can be replaced with one, in which case a sufficient condition for (2.4), and hence a sufficient condition for the black-white difference in North-South wage differentials to identify a lower bound on the black-white

[6] Another way to see this is to think of Assumption 2.2 as requiring that $E(y_l|l,r,x,v) = \tilde{\alpha}_{rx} + \tilde{\beta}_{rx}v = (\tilde{\alpha}_{rx} + \tilde{\beta}_{rx}\mu_{rx}) + (\tilde{\beta}_{rx}\sigma_{rx})\varepsilon = \alpha_{rx} + \beta_{rx}\varepsilon$, so that a one standard deviation increase in v increases southern wages less for a black worker than for an observably-identical white one.

[7] Because Assumption 2.3 places restrictions on the relationship between potentially counterfactual wages in the South and the unobserved determinants of the migration decision, it is not directly testable. However, a simple measure can provide some evidence on whether observed wages in the South are consistent with the assumption. To see this, note that under Assumption 2.2, counterfactual wages can be written $y_{lrx} = \alpha_{lrx} + \beta_{lrx}\varepsilon + u_{lrx}$, where u_{lrx} is uncorrelated with ε. Thus, $V(y|l=s,r,x) = \beta_{lrx}^2 V(\varepsilon|\varepsilon < -\mu_{rx}/\sigma_{rx}) + V(u_{lrx})$. Also note that, by Proposition 1 of Heckman and Honoré (1990), if the density function for ε is log concave (convex), i.e., $\log f_\varepsilon$ is concave (convex), then $V(\varepsilon|\varepsilon < \hat{\varepsilon})$ is increasing (decreasing) in $\hat{\varepsilon}$ and that a greater migration rate for blacks implies that $-\mu_{bx}/\sigma_{bx} < -\mu_{wx}/\sigma_{wx}$. Suppose for the moment that $V(u_{lbx}) = V(u_{lwx})$ for all l and x. In this case, if ε is log concave, $V(y|s,b,x) > V(y|s,w,x)$ implies that $\beta_{sbx} > \beta_{swx}$ (i.e., violations of the assumption are testable). Similarly, if ε is log convex, $V(y|s,b,x) < V(y|s,w,x)$ implies that $\beta_{sbx} < \beta_{swx}$ (i.e., the assumption itself is testable). When $V(u_{lrx})$ differs between blacks and whites, this heuristic test does not apply. However, concern that observed wages are highly contaminated with selection effects are implicitly concerns that the factors that contribute to migration are important determinants of wages, so that $V(u_{lrx})$ is small compared to $V(\varepsilon|\varepsilon < -\mu_{rx}/\sigma_{rx})$, which provides a justification for applying the heuristic test anyway.

difference in ATTs, is that

$$\frac{d}{d\hat{\varepsilon}}[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})] \geq 0 \tag{2.5}$$

on $\hat{\varepsilon} \in (-\mu_{bx}/\sigma_{bx}, -\mu_{wx}/\sigma_{wx})$. The identification of a lower bound on group difference in ATTs then becomes a question of whether (2.5) is likely to hold. The following result establishes two classes of distributions for which this property holds over some subset of the support. Moreover, and as I discuss below, the distributions used in standard theoretical and econometric models fall into one of these classes.

PROPOSITION 2.1. *Suppose that ε is distributed over $[L, \infty]$ with density f satisfying $\lim_{\varepsilon \to \infty} f = 0$. Then:*

1 If $E(\varepsilon|\varepsilon \geq \hat{\varepsilon})$ is convex and $E(\varepsilon|\varepsilon < \hat{\varepsilon})$ is concave, there exists an ε^* such that

$$\frac{d}{d\hat{\varepsilon}}[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})] \geq 0$$

for all $\hat{\varepsilon} \geq \varepsilon^*$. If f is symmetric, then ε^* is the mean. If the mean exceeds the median, then ε^* is less than the median.

2 If f is log convex and $f' \leq 0$ for all ε,

$$\frac{d}{d\hat{\varepsilon}}[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})] \geq 0$$

for all $\hat{\varepsilon}$.[8]

In Appendix 5, I show that the conclusions of Proposition 2.1 can also be derived from more fundamental restrictions on the underlying density functions (these restrictions, which amount to requiring that the log of the density becomes less concave as the density itself is decreases, are closely related to the log concavity condition that Heckman and Honoré, 1990 show is pivotal for identification of the canonical Roy model). However, the usefulness of the proposition hinges on whether it is reasonable to believe that the distributions of the unobserved determinants of counterfactual wages and migration belong to one of its classes. Figures 1 and 2 show that the distributions used in common theoretical and econometric models of sample selection, truncation, discrete choice, duration, and reliability fall into one of these classes. The first figure shows the left- and right-truncated expectations, and their difference, for parameterisations of the normal, logistic, uniform, gamma, Weibull, exponential and logistic distributions. For each of these distributions, the left-truncated expectation is convex, the right-truncated expectation is concave, and there is a point in the support beyond which the difference in truncated means is increasing. The second figure shows the same functions for (in some cases, different) parameterisations of the gamma, Weibull, Pareto, and lognormal distributions. With the exception of the lognormal, each of these distributions belongs to the second class defined in the proposition. For each distribution, the left- and right-truncated expectations are concave and the difference in truncated means is increasing on the entire support.[9]

[8] A function is log convex (concave) if its log is convex (concave).

[9] The lognormal density is neither log concave nor log convex on its entire support (Bagnoli and Bergstrom, 2005), and is not everywhere decreasing. Nevertheless, the difference in truncated means

The key implication of Proposition 2.1 is that the black-white difference in regional wage differences identifies a lower bound on the black-white difference in the causal effects of migration on wages (and the ATT itself for blacks if the ATT for whites is nonnegative) under distributional assumptions that are considerably weaker than those used in standard sample selection and other econometric models. If the net benefits ε of migrating are drawn from any distribution satisfying the conditions of the proposition, the racial difference in regional wage differences will identify a lower bound on the racial difference in ATTs as long as the group-specific migration thresholds $-\mu_{rx}/\sigma_{rx}$, $r \in \{b, w\}$, are sufficiently large or, equivalently, that the race- and covariate-specific probabilities of migrating are sufficiently low. A natural starting point for modelling the net benefits of migrating is to assume that they are drawn from a symmetric member of the first class of distributions given in Proposition 2.1 (for example, both a logit or probit model of the migration decision satisfy this assumption).[10] In this case, the proposition implies that the black-white difference in regional wage differentials bounds the racial difference in ATTs from below as long as $-\mu_{rx}/\sigma_{rx} \geq 0$ for $r \in \{b, x\}$—i.e., that less than half of southern-born blacks and whites with covariates x migrate to the North.[11]

If, instead, the distribution of the net migration benefits were drawn from a member of the first class of distributions that is right skewed in the (informal) sense that its mean exceeds its median (for example, if they follow a lognormal distribution), the lower-bound identification argument would apply at even smaller values of $-\mu_{rx}/\sigma_{rx}$, and thus migration rates in excess of .5. Indeed, if they were drawn from a member of the second class of distributions in Proposition 2.1, the lower-bound identification argument would apply at any migration rate. Consequently, a migration rate of .5 can be considered a conservative condition for the lower-bound identification argument.

3. BOUNDING THE WAGE EFFECTS OF THE GREAT MIGRATION

Although the results above recover bounds on the racial differences in ATTs conditional on a set of observed covariates, the identification argument can also be applied without covariates in order to bound the unconditional black-white difference in the causal effects of migration on wages.[12] Accordingly, I begin by using the identification results to obtain

for its second parameterisation is everywhere increasing, showing that the conditions of Proposition 2.1 are sufficient but not necessary.

[10] Strictly speaking, the ε represent 'standardised' benefits from migrating, although the actual benefits $v_{rx} = \mu_{rx} + \sigma_{rx}\varepsilon$ will inherit the convexity of ε. Also note that since log concavity is preserved under linear transformations, the v_{rx} will have the same log concavity as ε (Bagnoli and Bergstrom, 2005).

[11] Note that if race were replaced with time, the same argument could be applied to the temporal difference in differences $[E(y|l = 1, t = 1) - E(y|l = 0, t = 1)] - [E(y|l = 1, t = 0) - E(y|l = 0, t = 0)]$ to recover a lower bound on the change in the ATT over time. Along similar lines, if wages and migration were observed for both races at different time periods and the migration rate were stable for one of the races, the fuzzy difference-in-differences setup developed in de Chaisemartin and D'Haultfoeuille (2017) could be used to estimate the ATT itself (I thank an anonymous referee for making these observations).

[12] While conditioning on covariates is essential in research designs predicated on an assumption of selection on observables, in quasi-experimental research designs, this is often not the case. For example, in a typical difference-in-differences design that compares temporal changes in outcomes between the treatment and control groups, it is not necessary to condition on observed covariates as long as the pre-treatment evolution of outcomes for the treatment and control groups follow parallel paths. Under the identification results developed above, the situation is much the same. The lower-bound approach uses racial variation in the probability of migrating to bound the selection bias present in wage comparisons between migrant and non-migrant blacks. As long as the requirements for identification hold unconditionally, the procedure can be used to bound the group difference in ATTs without conditioning

population-average bounds on racial differences in the effect of northward migration on wages.

As Proposition 2.1 shows, as long as the race-specific migration rates are not too large, the selection bias component of the North-South wage differential will be decreasing in the migration probability, in which case a larger black migration rate implies the black-white difference in North-South wage differences will bound the black-white difference in ATTs from below. Following the discussion of the proposition, migration probabilities less than one half are likely sufficient to satisfy this requirement (this is conservative; if the distributions of the benefits from migrating are skewed right, the lower bound argument applies at even higher migration rates).

To provide evidence that the data are consistent with this condition, Figure 3 plots northward migration rates by year of birth for southern-born black and white men who, to avoid age effects, were at least 30 years old when surveyed. The data for this graph, and all further results in this section, are based on 1% Integrated Public Use Microdata Samples (IPUMS) of the 1940-1970 US Censuses (Ruggles et al., 2010), from which I include only southern-born black and white men aged 16-64.[13] For birth years prior to 1880, the sample sizes are small and the estimated migration rates are imprecise for men of both races. Past 1880, as the figure shows, black migration rates dominate white rates at all birth years, and exhibit a steeper trend. For example, a white man born in the South around 1940 had about a 20% chance of migrating to the North, while his black counterpart had about a 35% chance. Table 1 presents pooled and decade-specific linear models of the probability of migrating. On average between 1940 and 1970, the black migration rate of 23% was nearly twice as high as the white rate of 12%, with similar group differences within decades.

Figure 3 and Table 1 show that blacks migrated at significantly higher rates than whites, while both groups migrated with probability less than one half, suggesting that the data are consistent with the requirements of the identification argument. To apply the argument, in Table 2 I present regression estimates of the black-white difference in North-South log wage differences.[14] Consistent with previous evidence on the wage effects of the Great Migration, the regressions evince large wage differentials for migrant whites and even larger differentials for migrant blacks, both across and within decades. On average between 1940 and 1970, for example, white migrants' wages were about 28% greater than whites who remained in the South. For southern-born blacks, the North-South wage gap exceeded the white gap by an additional 36%. Applying the lower-bound identification argument, the difference-in-difference estimates in Table 2 (i.e., the coefficients on Black*North) suggest that, whatever effect it had on whites' wages, migration increased blacks' wages by at least an additional 36%. The decade and race-specific regional wage differentials are similar. In addition, under the hypothesis that the causal

on observed covariates (in this case, ε represents the contribution of unobserved factors to the net benefit of migrating as well as factors that are in principle observable).

[13] I classify states as southern using the Census Bureau's definition of the South: Alabama, Arkansas, Delaware, Florida, Georgia, Kentucky, Louisiana, Maryland, Mississippi, North Carolina, Oklahoma, South Carolina, Tennessee, Texas, Virginia and West Virginia are southern states. I define the North as any other state in the US. Although the Great Migration started well-before 1940, that decade is the first for which individual-level wage data are available.

[14] The wage measure consists of all income from wages and salary in the year before enumeration (this variable is named INCWAGE in the IPUMS dataset). The self-employed are included but business and farm income are not. All wages are inflated to 1999 dollars using the CPI weights supplied with the IPUMS. To focus on men working-age men, I restrict the sample to those aged 16-64.

effect of migration on whites' wages was, at worst, zero, these lower bounds on the black-white differences in ATTs can also be interpreted as lower bounds on the ATT itself for blacks. The implied bound of 36% agrees well with the first-differenced estimate, reported by Collins and Wanamaker (2014), of 63 log points for black men who migrated between 1910 and 1930, suggesting that the difference in differences recovers a (comfortingly) conservative lower bound on the average effect of migrating on black migrants' wages.

3.1. Covariate-specific effects

The unconditional difference-in-difference estimates presented in Table 2 can be interpreted as lower bounds on racial differences in the effects of migration on wages, averaged over both observed and unobserved determinants of wages and migration. To examine how racial differences in the wage effects of migration vary with respect to observed characteristics, I apply the identification argument conditional on several observed correlates of skill. As a starting point, I stratify the sample into men above and below 40 years old. Columns (1) and (2) of Table 3 replicate the migration-rate regressions for these subsamples (for simplicity, these regressions and the log wage regressions discussed below use pooled samples from all Census decades, although they include race-specific year fixed effects). The estimates show that, within these age strata, blacks migrated at higher rates than whites, but both groups migrated with probability less than one half. The corresponding columns (1) and (2) of Table 4 present age-group-specific difference-in-difference log wage regressions. The estimates imply that migration had a slightly greater impact on black-white relative wages for younger men (a 38% racial difference) than older men (a 33% difference), on average across the sample period. Column (3) of Table 3 shows that the required restrictions on migration rates also hold after conditioning on a full set of race- and year-specific indicators for membership in five-year age groups. The corresponding difference-in-difference log-wage estimate from Table 4 implies that the across-age-group average effect of migration on wages is 31% higher for blacks than for whites.[15]

To examine how education affects racial differences in the wage impacts of migration, I repeat the previous exercise for education, stratifying the sample into men with and without a high-school education. Columns (4) and (5) of Table 3 show that, within these educational strata, blacks were more likely to migrate, and the migration rate was less than one half for both groups. The difference-in-difference estimates in the corresponding columns of Table 4 imply that migration had a greater impact on the relative wages of less educated black men.[16]A potential caveat to this conclusion is that education may have been part of the mechanism through which migration increased wages, for example if higher educational quality or returns to education in the North

[15]The reason that this estimated average bound is smaller than the one presented in column (1) of Table 2 is that white migration rates are significantly smaller conditional on age (as Table 3 shows). Because, as Figure 1 illustrates, selection bias may decrease more rapidly with the migration probability when that probability is low, conditioning on age produces a more conservative lower bound on the black-white difference in ATTs. Note that, if the effect of migration varies with covariates, specification (3) of Table 4 identifies the racial difference in age-group specific average ATTs, weighted by the variance of a migration indicator conditional on the covariates (see Angrist and Krueger, 1999, Section 2.3).

[16]The estimates in column (6), which include a set of race- and year-specific educational attainment controls, can be interpreted as lower bounds averaged across all education strata. The educational attainment controls include indicators for having less than six, between six and eleven, exactly twelve, or greater than twelve years of education.

encouraged younger migrants to obtain more education (unfortunately the Census data do not identify the ages at which respondents migrated). In this case, conditioning on education may understate or overstate racial differences in the wage impacts of migration, depending on whether this mechanism is stronger for blacks or whites.

To explore occupational heterogeneity in the wage effects of the Great Migration, I repeat the above analysis after dividing the sample into those working in non-farm and farm occupations. Columns (7) and (8) of Table 3 show that the migration rate requirements are met within each of these subsamples. The corresponding columns of Table 4 suggest that the influence of migration on the wages of blacks relative to whites was about the same for those in non-farm and farm occupations. However, since regional differences in occupations are a potentially important channel through which migration increased wages, the comparisons that these regressions perform are not ideal for understanding racial differences in the effects of migration on wages. Illustrating this logic, column (9) of Table 4 presents difference-in-difference wage regressions that include race- and year-specific occupation and industry controls.[17] The resulting difference-in-difference estimate is substantially smaller than those for the previous specifications, implying that changing the occupations and industries in which blacks were employed was an important part of how the Great Migration increased their wages relative to whites (of course, the estimates remain tacit about the extent to which these effects are a consequence of industrial and occupational variation in productivity, racial discrimination, or other factors).[18]

3.2. Comparison with point-identification approaches

For the purpose of comparison with the Roy-model bounds discussed above, I also present race-specific point estimates of the effect of Northward migration on wages, obtained using standard point-identification methods. A natural starting point is to compare the results of the lower-bound approach to those obtained under an unconfoundedness assumption (Rubin, 1974), which holds that counterfactual wages in the North and South are independent of migration status conditional on a set of observed covariates. A practical challenge to using unconfoundedness to identify the effect of migration on wages during the Great Migration period is that the choice of conditioning variables is not obvious. While concern that migrants are selected in terms of unobserved skill might suggest conditioning on observed indicators of skill such as education or occupation, as I discuss above, these variables may themselves be outcomes of the migration decision. Arguably the more serious challenge is the possibility that the unconfoundedness assumption is violated by selection into migration on the basis of factors not observed in the data that also affect counterfactual wages. Keeping these challenges in mind, for the purpose of this exercise I assume that migration and wages are unconfounded conditional on education and age.

I use an inverse probability weighting approach to estimating the effect of migration

[17] These controls consist of indicators for membership in one-digit occupation groups defined as int(OCC1950/100) and int(IND1950/100) where int(x) is the largest integer less than x and OCC1950 and IND1950 are 1950 occupation and industrial classifications provided by the IPUMS Census samples.

[18] Note that, as column (9) of Table 3 shows, conditioning on occupation and industry does not substantially reduce the estimated migration rate for whites, implying that the lower difference-in-difference estimate is not a consequence of more selective migration among whites within occupational and industrial strata.

on wages under unconfoundedness. To implement this approach, I estimate race-specific propensity scores using logit models of Northward migration as functions of the full sets of indicators for age and educational attainment detailed above (as well as year effects for the pooled samples of all Census decades). I then regress observed wages in the North and South on age and education (and, for the pooled samples, year) effects, weighting each untreated observation by the ratio of its propensity for being treated to its propensity for being untreated. The ATTs are estimated as the implied difference between wages in the North and South, averaged according to the covariate distribution for migrants. The motivation for this doubly-robust estimator is that it will consistently estimate the average effect of the treatment on the treated if either the regression function or the propensity score is correctly specified (see Hahn, 1998 for reweighting estimates of the ATT and Bang and Robins, 2005 for a discussion of doubly robust estimators).

The resulting estimates are presented in the top panel of Table 5. I focus on pooled estimates, though the table also includes decade-specific effects. The inverse-probability-weighted results imply that northward migration increased wages by about 43 log points for blacks and 18 log points for whites. The corresponding unconditional OLS point estimates from Table 2 are significantly larger, at about 64 log points for blacks and 28 for whites (with a difference in differences of 36). Viewed as a lower bound on the effects of the Great Migration on blacks' wages, the difference in difference estimate of .36 presented in Table 2 compares favourably to the weighted estimate of .43 (as do the across-covariate averages in Table 4), supporting the idea that the Roy-model argument identifies a lower bound. The identification results presented above also imply that the difference in difference estimate of .43 can be interpreted as a lower bound on the black-white difference in ATTs, which exceeds the black-white difference in weighted regression estimates of .25. However, while an obvious explanation for why the inverse-probability-weighted point estimates from Table 5 are smaller than the OLS estimates from Table 2 is that migrants were selected on age and education, another is that migration encouraged educational attainment. If, as intuition suggests, the effect of migration on educational attainment were stronger for blacks, conditioning on education in order to estimate the effect of migration under unconfoundedness would understate the ATT for blacks as well as the black-white difference in ATTs, in which case the lower-bound approach would be preferable.[19]

For further comparison, I also estimate the effects of migration on wages using a parametric switching regression model of wages and the net benefit from migrating. To implement this estimator, I assume that wages and the utility gain from migrating are linear functions of age and education effects and (joint normally distributed) disturbance terms. For simplicity, I also assume that age and education enter the wage function independently of location, so that the effect of migration on wages is represented by a simple intercept shift.[20] In principle, an advantage of the switching regression approach is that it relaxes the assumption of unconfoundedness by allowing for the possibility that the disturbances in the wage and migration equations are correlated. In practice, this flexibility poses its own challenges. Switching regression (and related) models are typically estimated under the assumption that there is a regressor in the selection equation that

[19]This explanation is also consistent with how including covariates affects the difference-in-difference estimates in Table 3.

[20]That is, I assume that the log wage for an individual with education level e belonging to age group a is $\ln w = \beta_0 + \beta_e + \beta_a + \beta_1 m + u$, where m is an indicator for migrating, $m = 1(\gamma_0 + \gamma_e + \gamma_a + \varepsilon > 0)$, and (u, ε) are drawn from a bivariate normal distribution.

can be excluded from the outcome equations. In the Great Migration context, it is difficult to argue that any of the potential observable covariates satisfy this exclusion restriction (if they did, the wage effect could be estimated by instrumental variables). However, the model is identified even absent such an exclusion restriction, although in this case identification is derived from distributional assumptions alone and, as I show below, the resulting estimates appear to be sensitive to these assumptions.

The switching regression estimates are presented in the bottom panel of Table 5. The point estimates of the coefficients on North imply that migrating North increased wages by 92 and 36 log points for blacks and whites, respectively, on average between 1940 and 1970. These estimates are considerably higher than the corresponding simple OLS estimates given in Table 2. Taken at face value, this comparison suggests negative selection into migration among both blacks and whites—that low-counterfactual-wage individuals were more likely to migrate, leading observed regional wage differentials to understate the causal effect of migration on wages. However, the estimated migration effects vary wildly across both racial groups and time, calling the specification and identification of the model into question. In contrast, the Roy-model bounds presented previously, which are also identified without an exclusion restriction but do not impose strong distributional assumptions, are stable over time and broadly consistent with both intuition and the effects estimated under the unconfoundedness assumption.

4. CONCLUSION

It has long been known that the Great Migration played an important role in black relative economic progress during the 20th century. However, concerns about the possibility of selective migration have called into question the causal interpretation of North-South wage differentials, as well as racial differences in those differentials. To provide new evidence on the causal effects of northward migration on blacks' wages and black-white wage convergence, this paper combines racial differences in migration rates with a Roy model of migration and counterfactual wages to interpret black-white differences in North-South wage differentials as lower bounds on black-white differences in the causal effects of migration on wages. The idea behind the identification argument is that more selective migration among whites implies greater selection bias in regional wage comparisons for whites, in which case subtracting regional wage differentials from whites from those for blacks over-controls for the selection bias present in the black differentials, bounding the racial difference in causal effects from below. Moreover, under the *a posteriori* plausible hypothesis that migration did not decrease whites' wages, the black-white difference in wage impacts is itself a conservative lower bound on the average effect of migration on black migrants' wages.

These identification results, coupled with estimates of the racial difference in regional wage differentials, imply that northward migration increased blacks' wages by at least 36% more than whites', on average between 1940 and 1970, and thus that migration increased the typical black migrant's wage by at least 36%. Covariate-specific estimates suggest that these effects were slightly larger for younger and less educated blacks, although the magnitudes of the effects are large across all covariate strata. Conditioning on indicators for occupation and industry substantially reduce the estimated black-white difference, suggesting that regional differences in industrial and occupational composition were an important mechanism through which migration affected wages.

The estimated lower bounds compare favourably to point estimates of the effect of migration on wages estimated via inverse-probability-of-treatment-weighted regressions motivated by an unconfoundedness assumption and via a switching regression model of migration and wages, while circumventing the challenges associated with applying these estimators. Although in principle these alternative approaches can point-identify the treatment effects of interest, the requirements for their validity may not hold in some applications. In the context of the Great Migration, it is unclear which, if any, of the available variables should be included in the conditioning set for an identification strategy based on unconfoundedness or used as exclusion restrictions for a switching-regression approach.

While the theoretical and empirical results in this paper redouble the evidence that migration increased blacks' wages (both absolutely and relative to whites), the underlying mechanisms through which it increased them remains a largely open question. Although it is intuitive that regional differences in occupational and industrial composition are an important channel through which migration affected wages, it is not obvious why this channel matters differently for blacks and whites. Moreover, the remainder of the wage effect may be a consequence of reduced discrimination in the North, race-specific productivity effects, racial differences in the distribution of migrants throughout the North, or other factors. Although it is beyond the scope of the current paper and the reach of its methods, attempting to answer these questions suggests interesting and potentially important avenues for future work on the Great Migration.

ACKNOWLEDGEMENTS

I thank Robert Miller, Lowell Taylor, John Conlon, several anonymous referees, the editor, and seminar participants at Carnegie Mellon University, The University of Mississippi, and the Western Economics Association annual conference for their insights.

REFERENCES

Amemiya, T. (1984). Tobit Models: A Survey. *Journal of Econometrics 24*(81), 3–61.

An, M. Y. (1995). Log-concave Probability Distributions: Theory and Statistical Testing. Working Paper 919.

Angrist, J. D. and A. B. Krueger (1999). Empirical Strategies in Labor Economics. In O. Ashenfelter and D. Card (Eds.), *Handbook of Labor Economics*, Volume 3, Chapter 23, pp. 1277–1366. Elsevier Science B.V.

Arabmazar, A. and P. Schmidt (1982). An Investigation of the Robustness of the Tobit Esitmator to Non-Normality. *Econometrica 50*(4), 1055–1063.

Bagnoli, M. and T. Bergstrom (2005). Log-Concave Probability and its Applications. *Economic Theory 26*(2), 445–469.

Bang, H. and J. M. Robins (2005). Doubly Robust Estimation in Missing Data and Causal Inference Models. *Biometrics 61*, 962–972.

Borell, C. (1975). Convex Set Functions in d-Space. *Periodica Mathematica Hungarica 6*, 111–136.

Caplin, A. and B. Nalebuff (1991). Aggregation and Social Choice: A Mean Voter Theorem. *Econometrica 59*(1), 1–23.

Collins, W. J. and M. H. Wanamaker (2014). Selection and Economic Gains in the Great Migration of African Americans: New Evidence from Linked Census Data. *American Economic Journal: Applied Economics 6*(1), 220–252.

de Chaisemartin, C. and X. D'Haultfoeuille (2017). Fuzzy Differences in Differences. *Review of Economic Studies*, Forthcoming.

D'Haultfoeuille, X. and A. Maurel (2013a). Another Look at the Identification at Infinity of Sample Selection Models. *Econometric Theory 29*(1), 213–224.

D'Haultfoeuille, X. and A. Maurel (2013b). Inference on an Extended Roy Model, with an Application to Schooling Decisions in France. *Journal of Econometrics 174*(2), 95–106.

Donohue, J. and J. Heckman (1991). Continuous Versus Episodic Change: The Impact of Civil Rights Policy on the Economic Status of Blacks. *Journal of Economic Literature 29*(4), 1603–1643.

Eisenhauer, P., J. Heckman, and E. Vytlacil (2015). Generalized Roy Model and the Cost-Benefit Analysis of Social Programs. *Journal of Political Economy 123*(2), 413–443.

Hahn, J. (1998). On the Role of the Propensity Score in Efficient Semiparametric Estimation of Average Treatment. *Econometrica 66*(2), 315–331.

Head, K. (2013). Skewed and Extreme: Useful Distributions for Economic Heterogeneity. Working paper.

Heckman, J. J. (1976). The Common Structure of Statistical Models of Truncation, Sample Selection and Limited Dependent Variables and a Simple Estimator for Such Models. *Annals of Economic and Social Measurement 5*(4), 475–492.

Heckman, J. J. (1979). Sample Selection Bias as a Specification Error. *Econometrica 47*(1), 153–161.

Heckman, J. J. and B. E. Honoré (1990). The Empirical Content of the Roy Model. *Econometrica 58*(5), 1121–1149.

Heckman, J. J., S. Urzua, and E. Vytlacil (2006). Understanding Instrumental Variables in Models with Essential Heterogeneity. *Review of Economics and Statistics 88*(3), 389–432.

Jawitz, J. W. (2004). Moments of Truncated Continuous Univariate Distributions. *Advances in Water Resources 27*(3), 269–281.

Manski, C. F. (1989). Anatomy of the Selection Problem. *The Journal of Human Resources 24*(3), 343.

Manski, C. F. (1990). Nonparametric Bounds on Treatment Effects. *The American Economic Review 80*(2), 319–323.

Manski, C. F. (2005). *Social Choice with Partial Knowledge of Treatment Response.* Princeton University Press.

Mares, V. and J. M. Swinkels (2014). On the Analysis of Asymmetric First Price Auctions. *Journal of Economic Theory 152*, 1–40.

Prékopa, A. (1971). Logarithmic Concave Measures with Application to Stochastic Programming. *Acta Sci. Math.(Szeged) 32*, 301–316.

Prékopa, A. (1973). On Logarithmic Concave Measures and Functions. *Acta Sci. Math.(Szeged) 34*, 335–343.

Roy, A. (1951). Some Thoughts on the Distribution of Earnings. *Oxford Economic Papers 3*(2), 135–146.

Rubin, D. B. (1974). Estimating Causal Effects of Treatments in Randomized and Nonrandomized Studies. *Journal of Educational Psychology 66*(5), 688–701.

Ruggles, S., J. T. Alexander, K. Genadek, R. Goeken, M. B. Schroeder, and M. Sobek (2010). Integrated Public Use Microdata Series: Version 5.0 [machine-readable database]. *Minneapolis: University of Minnesota.*

Smith, J. and F. Welch (1989). Black Economic Progress after Myrdal. *Journal of Economic Literature 27*(2), 519–564.

Tobin, J. (1958). Estimation of Relationships for Limited Dependent Variables. *Econometrica 26*(1), 24–36.

Tolnay, S. E. (2003). The African American "Great Migration" and Beyond. *Annual Review of Sociology 29*(1), 209–232.

Wooldridge, J. M. (2002). *Econometric Analysis of Cross Section and Panel Data* (1st ed.). MIT Press.

5. APPENDIX A: PROOF OF PROPOSITION 2.1

Proof of Proposition 2.1: To prove the first part, note first that since $E(a|a \geq \hat{a}) - E(a| < \hat{a})$ is convex by assumption, if this difference is increasing at L, it is increasing on the entire support. Otherwise, suppose that $\lim_{\hat{a}\to\infty} dE(a|a < \hat{a})/d\hat{a} > 0$. Then, since a has infinite support, there exists an $\hat{\varepsilon}$ such that $E(\varepsilon|\varepsilon \leq \hat{\varepsilon}) > E(\varepsilon)$, a contradiction. Thus $\lim_{\hat{\varepsilon}\to\infty} dE(\varepsilon|\varepsilon < \hat{\varepsilon})/d\hat{\varepsilon} = 0$, and since $dE(\varepsilon|\varepsilon \geq \hat{\varepsilon})/d\hat{\varepsilon} \geq 0$, there is a unique ε^* at which $d[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})]/d\hat{\varepsilon} = 0$ and the difference in truncated means is minimised.

Next, write

$$\frac{d}{d\hat{\varepsilon}}[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})] = \frac{f(\hat{\varepsilon})}{1-F(\hat{\varepsilon})}\left(\frac{\int_{\hat{\varepsilon}}^{\infty} tf(t)dt}{1-F(\hat{\varepsilon})} - \hat{\varepsilon}\right) - \frac{f(\hat{\varepsilon})}{F(\hat{\varepsilon})}\left(\hat{\varepsilon} - \frac{\int_{L}^{\hat{\varepsilon}} f(t)dt}{F(\hat{\varepsilon})}\right).$$

At the median, $\tilde{\varepsilon}$, of ε, this expression becomes $4f(\tilde{\varepsilon})\,[E(\varepsilon) - \tilde{\varepsilon}]$. Thus, for f symmetric, $\varepsilon^* = E(\varepsilon) = \tilde{\varepsilon}$. Instead, if $E(\varepsilon) > \tilde{\varepsilon}$, $d[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})]/d\hat{\varepsilon} > 0$ at $\tilde{\varepsilon}$, so $\varepsilon^* \leq \tilde{\varepsilon}$.

To prove the second part, note that $f(\varepsilon)$ log convex with $\lim_{\varepsilon\to\infty} f = 0$ implies that $1-F$ is log convex and that $f' \leq 0$ for all ε implies that F is log concave (Bagnoli and Bergstrom, 2005). But $1-F$ log convex implies $dE(\varepsilon|\varepsilon \geq \hat{\varepsilon})/d\hat{\varepsilon} \geq 1$ while F log concave implies $dE(\varepsilon|\varepsilon < \hat{\varepsilon})/d\hat{\varepsilon} \leq 1$ (see, e.g., Heckman and Honoré, 1990). Thus $d[E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - E(\varepsilon|\varepsilon < \hat{\varepsilon})]/d\hat{\varepsilon} \geq 0$ for all $\hat{\varepsilon}$. □

APPENDIX B: THE CONVEXITY OF TRUNCATED EXPECTATIONS

A comparison of Figures 1 and 2 shows that distributions with log concave densities tend to belong to the first class of distributions defined in Proposition 2.1 while those with log convex densities tend to belong the the second class. For example, the normal, logistic, and exponential densities are log concave, as are the gamma and Weibull densities when their shape parameters exceed one. Figure 1 shows that these distributions have convex (concave) right- (left-) truncated expectations. Similarly, the Pareto density is log convex, as are the gamma and Weibull when their shape parameters are less than one. Figure 2 shows that these distributions have concave left- and right-truncated expectations. The uniform and lognormal densities lie somewhere between these extremes; the uniform density is both log concave and log convex while the lognormal density switches from log concave to log convex.

Proposition A.1 shows that this pattern is not coincidental; while the log concavity of the density is not sufficient for the convexity of the truncated expectation, these properties are closely related.

Proposition A.1. *Suppose that ε is distributed over $[L, H]$ with density f and*

$$\frac{d}{d\varepsilon}\left|\frac{[\log f(\varepsilon)]''}{\left\{[\log f(\varepsilon)]'\right\}^2}\right| \lesseqgtr 0 \quad \textit{when} \quad f'(\varepsilon) \lesseqgtr 0. \tag{A.1}$$

Then:

1 If f is log concave and $\lim_{\varepsilon \to L} f = \lim_{\varepsilon \to H} f = 0$, $E(\varepsilon|\varepsilon \geq \hat{\varepsilon})$ is convex and $E(\varepsilon|\varepsilon < \hat{\varepsilon})$ is concave.
2 If f is log convex and $\lim_{\varepsilon \to H} f = 0$, $E(\varepsilon|\varepsilon \geq \hat{\varepsilon})$ is concave.

Noting its similarity to the measure of absolute risk aversion, what condition (A.1) requires is that the log of the density become less concave as the density itself decreases.[21] To illustrate the proposition, consider first the standard normal and logistic distributions, both of which have log concave densities and, as Figure 1 shows, convex left- and concave right-truncated expectations. The expression $|(\log f)''/(\log f)'^2|$ evaluates to $1/\varepsilon^2$ for the normal density and $2\exp(\varepsilon)/[1-\exp(\varepsilon)]^2$ for the logistic density; both of these functions are decreasing on $\varepsilon > 0$. For the log convex Pareto density with shape parameter β, this expression is $1/(\beta+1)$, which does not depend on ε; the left-truncated expectation is linear.

The proof of Proposition A.1 relies on an extension of the Prkopa-Borell theorem (Prékopa, 1971, 1973; Borell, 1975) due to Mares and Swinkels (2014).[22] Define the local ρ-concavity of $g(c)$ at c by

$$\rho_g(c) = 1 - \frac{g(c)g''(c)}{[g'(c)^2]}.$$

The justification for this definition is that if the local ρ-concavity of $g(c)$ at c is t, then g^t/t is linear at c. In showing that the local ρ-concavity of g can be used to bound the local ρ-concavity of the function $\bar{G}(c) = \int_c^1 g(t)dt$, Mares and Swinkels (2014, Lemma 3) provide the following lemma for an arbitrary, positive function g on the unit interval.

Lemma A.1. (Mares and Swinkels, 2014) *If $g(0) = 0$ and ρ_g is monotone on some interval $[0, \hat{c}]$, then $\rho_{\int_0^c g(s)ds}$ and $\rho_g(c)$ share the same monotonicity on $[0, \hat{c}]$. If $g(1) = 0$ and ρ_g is monotone on $[\hat{c}, 1]$, then $\rho_{\int_c^1 g(s)ds}$ and ρ_g share the same monotonicity on $[\hat{c}, 1]$.*

Proof of Proposition A.1: For the log concave case, I prove the result for $E(\varepsilon|\varepsilon \geq \hat{\varepsilon})$. The convexity of $-E(\varepsilon|\varepsilon < \hat{\varepsilon})$ follows by analogy. First, note that, since $E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) - \hat{\varepsilon} = [\int_{\hat{\varepsilon}}^H 1 - F(t)dt]/[1 - F(\hat{\varepsilon})]$ (this follows from integration by parts, see Bagnoli and Bergstrom, 2005), we can write

$$\frac{d}{d\hat{\varepsilon}} E(\varepsilon|\varepsilon \geq \hat{\varepsilon}) = \frac{f(\hat{\varepsilon})}{1 - F(\hat{\varepsilon})} \frac{\int_{\hat{\varepsilon}}^H 1 - F(t)dt}{1 - F(\hat{\varepsilon})}.$$

Since

$$\rho_{\int 1-F}(\hat{\varepsilon}) = 1 - \frac{f(\hat{\varepsilon}) \int_{\hat{\varepsilon}}^H 1 - F(t)dt}{[1 - F(\hat{\varepsilon})]^2},$$

$E(\varepsilon|\varepsilon \geq \hat{\varepsilon})$ convex is equivalent to $\rho'_{\int_{\hat{\varepsilon}}^H 1-F}(\hat{\varepsilon}) \leq 0$. By Lemma A.1, $\rho'_{1-F}(\hat{\varepsilon}) \leq 0$ implies

[21] For an increasing utility function u, $-u''/u'$ will be decreasing if $-u''/(u')^2$ is (though the risk aversion measure has to be renormalised when applied to log densities, which are not monotone increasing). The proof relies on the concept, due to Mares and Swinkels (2014), of local ρ-concavity, which those authors show is closely related to risk aversion. Note that there is no condition for the right-truncated expectation in the log convex case because the curvature of this expectation is determined by the log concavity of the distribution function, and many log convex densities have log concave distribution functions.
[22] See Caplin and Nalebuff (1991) for an introduction to ρ-concavity and the Prkopa-Borell theorem.

$\rho'_{\int_{\hat\varepsilon}^{H} 1-F}(\hat\varepsilon) \leq 0$. Because log concave densities are unimodal (see An, 1995), $\rho_{1-F}(\hat\varepsilon)' \leq 0$ whenever $\hat\varepsilon$ is less than or equal to the mode of ε, since

$$\rho_{1-F}(\hat\varepsilon) = 1 - \frac{[-f'(\hat\varepsilon)][1-F(\hat\varepsilon)]}{[f(\hat\varepsilon)]^2} = 1 + \frac{f'(\hat\varepsilon)[1-F(\hat\varepsilon)]}{[f(\hat\varepsilon)]^2}$$

and, when $f' > 0$, f'/f and $(1-F)/f$ are positive and, by log concavity, they are always decreasing.

When $\hat\varepsilon$ exceeds the mode, so that $f' < 0$, we can apply Lemma A.1 once again in order to infer the sign of $\rho'_{1-F}(\hat\varepsilon)$ from that of $\rho'_f(\hat\varepsilon)$. Noting that, since f is log concave, it can be written $f(\hat\varepsilon) = \exp[h(\hat\varepsilon)]$ where h is a concave function,

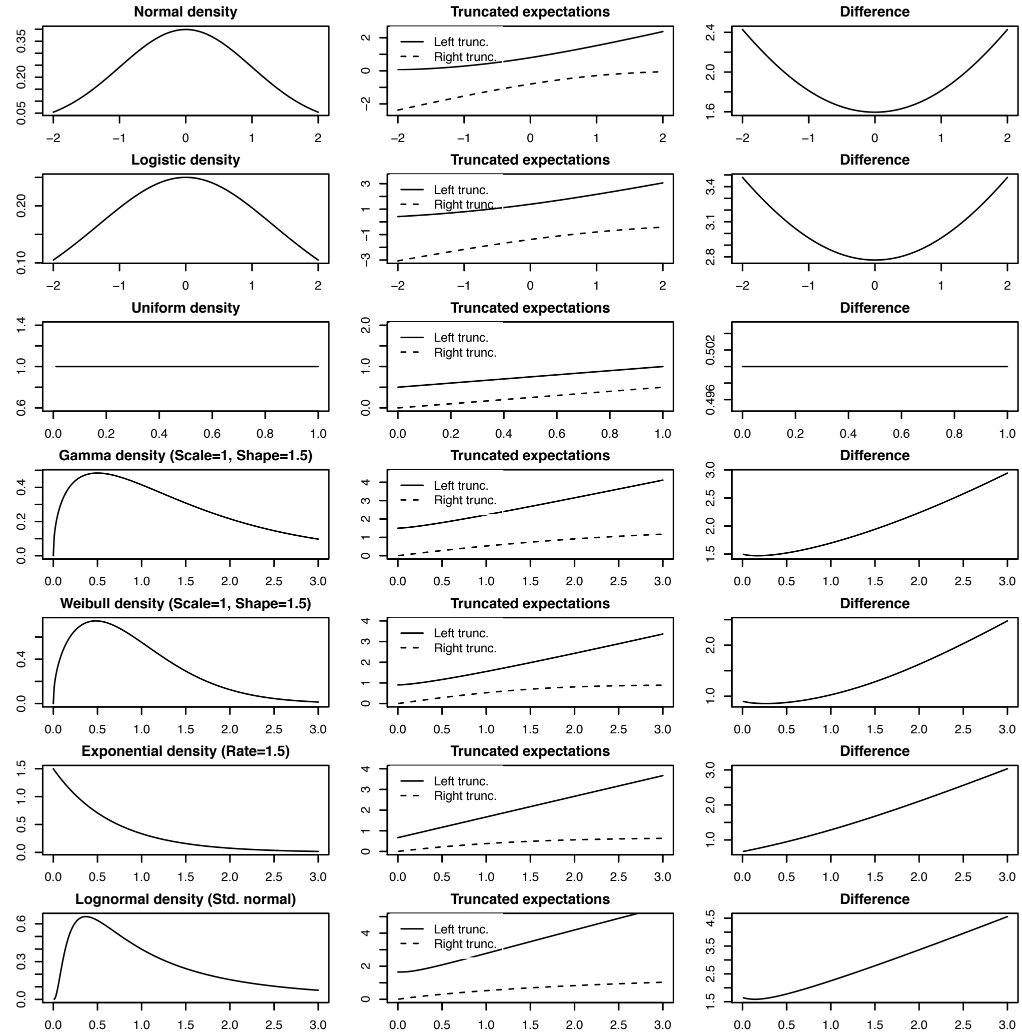
$$\rho_f(\hat\varepsilon) = 1 - \frac{f''(\hat\varepsilon)f(\hat\varepsilon)}{[f'(\hat\varepsilon)]^2} = 1 - \frac{\exp[h(\hat\varepsilon)]\left\{\exp[h(\hat\varepsilon)]h'(\hat\varepsilon)^2 + \exp[h(\hat\varepsilon)]h''(\hat\varepsilon)\right\}}{\left\{\exp[h(\hat\varepsilon)]h'(\hat\varepsilon)\right\}^2} = -\frac{h''(\hat\varepsilon)}{[h'(\hat\varepsilon)]^2}.$$

Since log concavity implies $h'' < 0$, under the conditions of the proposition, $-h''/(h')^2$ is positive and (weakly) decreasing, so $\rho_f(\hat\varepsilon)' \leq 0$, implying that $\rho_{1-F}(\hat\varepsilon)' \leq 0$ and hence $\rho_{\int_{\hat\varepsilon}^{H} 1-F}(\hat\varepsilon)' \leq 0$, establishing the result.

For the log convex case, note that if f is log convex then f'/f is increasing and, since $f(H) = 0$ implies that $1-F$ is also log convex (see Theorem 2 of Bagnoli and Bergstrom, 2005), $(1-F)/f$ is increasing as well. Thus, $\rho_{1-F}(\hat\varepsilon)$, and consequently $\rho_{\int_{\hat\varepsilon}^{H} 1-F}(\hat\varepsilon)$ are positive and increasing when $f' > 0$. When $f' < 0$, by the conditions of the proposition, we have $h''/(h')^2$ positive and decreasing, so that $\rho_f(\hat\varepsilon)$, $\rho_{1-F}(\hat\varepsilon)$ and hence $\rho_{\int_{\hat\varepsilon}^{H} 1-F}(\hat\varepsilon)$ are increasing, implying that $E(\varepsilon|\varepsilon \geq \hat\varepsilon)$ is concave. □

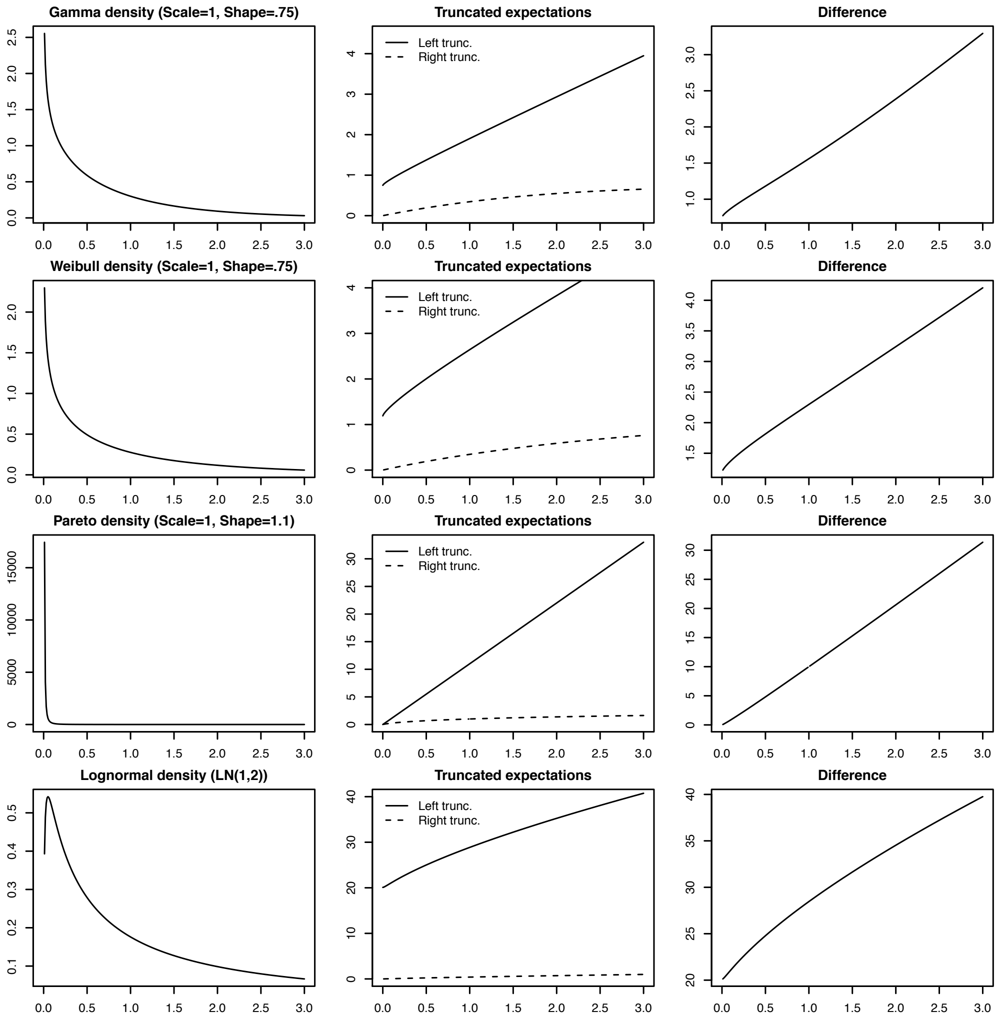
FIGURES AND TABLES

Figure 1. Distributions with convex (concave) left- (right-) truncated expectations



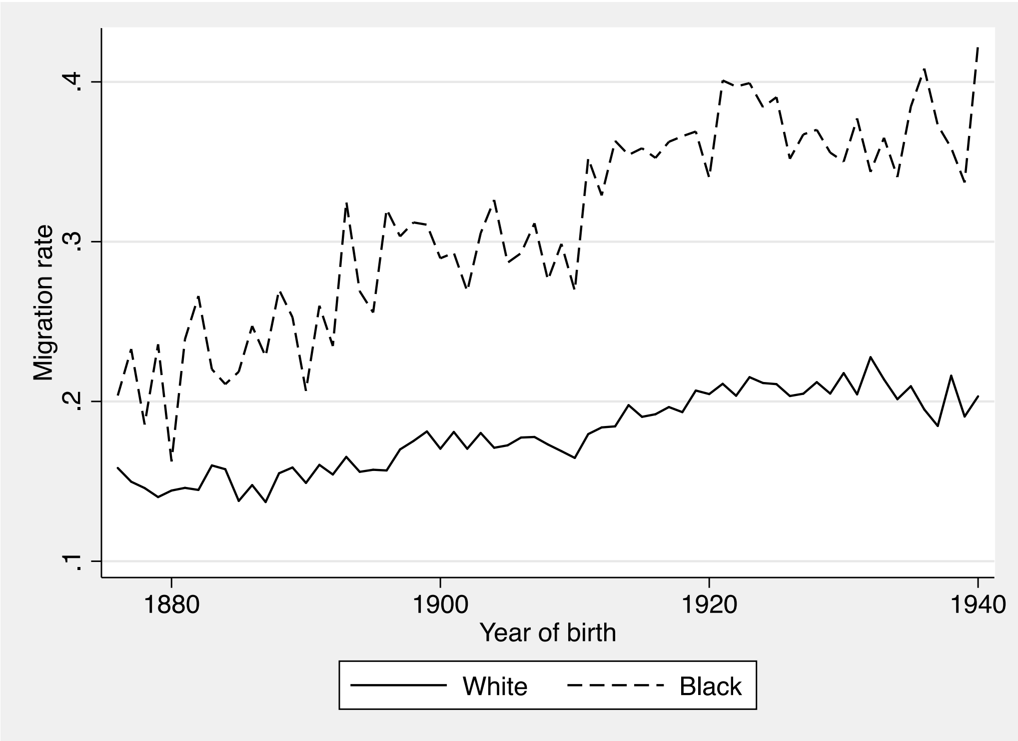
Notes—Difference denotes $E(a|a \geq \hat{a}) - E(a|a < \hat{a})$. Density formulae taken from Bagnoli and Bergstrom (2005). Expressions for the left- and right-truncated moments of the normal, logistic, gamma, Weibull and lognormal densities can be found in Arabmazar and Schmidt (1982), Heckman and Honoré (1990), and Jawitz (2004). By direct calculation, if $a \sim U[0,1]$ then $E(a|a \geq \hat{a}) = 1/2 + \hat{a}/2$ and $E(a|a < \hat{a}) = \hat{a}/2$. If a is exponential with rate parameter λ, then it can be shown (integrate by parts and apply L'Hpital's rule) that $E(a|a \geq \hat{a}) = 1/\lambda + \hat{a}$ and $E(a|a < \hat{a}) = 1/\lambda - \hat{a}/(e^{\lambda\hat{a}} - 1)$ (see also Head, 2013).

Figure 2. Distributions with concave truncated expectations



Notes—Difference denotes $E(a|a \geq \hat{a}) - E(a|a < \hat{a})$. Density formulae taken from Bagnoli and Bergstrom (2005). Expressions for the left- and right-truncated moments of the gamma, Weibull and lognormal densities can be found in Jawitz (2004). If a is Pareto distributed with shape parameter β then it can be shown that $E(a|a \geq \hat{a}) = \beta\hat{a}/(\beta - 1)$ and $E(a|a < \hat{a}) = [\beta/(\beta - 1)](1 - \hat{a}^{1-\beta})/(1 - \hat{a}^{-\beta})$ (see also Head, 2013).

Figure 3. Migration rates by year of birth



Notes—Probability of living in the North, by birth year, for black and white men, aged 30 or later and born after 1850.

Table 1. Migration rates

	All decades	1940	1950	1960	1970
Black	0.108***	0.0569***	0.120***	0.130***	0.126***
	(0.00685)	(0.00749)	(0.00993)	(0.00872)	(0.00810)
Constant	0.117***	0.130***	0.166***	0.193***	0.187***
	(0.00733)	(0.00736)	(0.00944)	(0.0105)	(0.0100)
Observations	517,361	133,059	45,130	162,833	176,339

Notes—Dependent variable is an indicator for living in the North. Pooled models include decade effects. Sample consists of southern-born men greater aged 16-64 with nonzero wages. Standard errors clustered on state-year of birth. *** p<0.01, ** p<0.05, * p<0.1.

Table 2. Difference-in-difference regressions

	All decades	1940	1950	1960	1970
Black		-0.684***	-0.587***	-0.667***	-0.532***
		(0.0153)	(0.0219)	(0.0183)	(0.0177)
North	0.282***	0.368***	0.306***	0.282***	0.236***
	(0.0210)	(0.0274)	(0.0327)	(0.0368)	(0.0421)
Black*North	0.362***	0.325***	0.400***	0.388***	0.338***
	(0.0147)	(0.0243)	(0.0335)	(0.0257)	(0.0249)
Observations	384,818	83,198	32,479	125,706	143,435

Notes—Dependent variable is the log annual wage. Pooled models include race-specific decade effects. Sample consists of southern-born men greater aged 16-64 with nonzero wages. Wage is defined as all income from wages in the year before enumeration. Standard errors clustered on state-year of birth. *** p<0.01, ** p<0.05, * p<0.1.

Table 3. Migration rates (with covariates)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Black	0.0836***	0.145***	0.109***	0.0958***	0.212***	0.132***	0.118***	0.0702***	0.103***
	(0.00699)	(0.00930)	(0.00671)	(0.00682)	(0.0102)	(0.00703)	(0.00725)	(0.00585)	(0.00679)
Constant	0.108***	0.139***	0.0462***	0.115***	0.153***	0.0373***	0.143***	0.0553***	0.0987***
	(0.00944)	(0.0118)	(0.0136)	(0.00706)	(0.0109)	(0.00533)	(0.00893)	(0.00457)	(0.0102)
Sample	≤40	>40	All	≤HS	>HS	All	Non-farm	Farm	All
Age FEs			Y						
Education FEs						Y			
Occupation and industry FEs									Y
Observations	306,848	210,513	517,361	426,566	90,795	517,361	429,006	88,355	465,103
R-squared	0.017	0.025	0.027	0.018	0.036	0.029	0.018	0.019	0.055

Notes—Dependent variable is an indicator for living in the North. Pooled data for 1940-1970. Sample consists of southern-born men greater aged 16-64 with nonzero wages. All models include decade effects. Age effects consist of a set of indicators for five-year age groups. Education effects consist of a set of indicators for having less than 6 years of schooling, between 6 and 11 years of schooling, 12 years of schooling, or more than 12 years of schooling. Occupation and industry effects are indicators for membership in one-digit industry and occupation codes (defined as the largest integer less than the original three-digit 1950 industry and occupational classifications divided by 100). $\gtreqless$ 40 denote indicators for being above or below 40 years old at the time of enumeration. HS is an indicator for having 12 years of schooling. Standard errors clustered n state-year of birth. *** p<0.01, ** p<0.05, * p<0.1.

Table 4. Difference-in-difference regressions (with covariates)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
North	0.282***	0.253***	0.212***	0.328***	0.109***	0.261***	0.260***	0.469***	0.277***
	(0.0285)	(0.00982)	(0.00800)	(0.0232)	(0.0189)	(0.0213)	(0.0211)	(0.0412)	(0.0166)
Black*North	0.377***	0.327***	0.307***	0.343***	0.311***	0.345***	0.362***	0.364***	0.184***
	(0.0192)	(0.0138)	(0.00998)	(0.0165)	(0.0203)	(0.0145)	(0.0147)	(0.0631)	(0.0116)
Sample	≤40	>40	All	≤HS	>HS	All	Non-farm	Farm	All
Age FEs			Y						
Education FEs						Y			
Occupation and industry FEs									Y
Observations	233,320	151,498	384,818	308,370	76,448	384,818	370,305	14,513	374,807
R-squared	0.171	0.260	0.419	0.193	0.183	0.242	0.195	0.245	0.388

Notes—Dependent variable is the log annual wage. Pooled data for 1940-1970. Sample consists of southern-born men greater aged 16-64 with nonzero wages. All models include race-specific decade effects. Age effects consist of a set of indicators for five-year age groups. Education effects consist of a set of indicators for having less than 6 years of schooling, between 6 and 11 years of schooling, 12 years of schooling, or more than 12 years of schooling. Occupation and industry effects are indicators for membership in one-digit industry and occupation codes (defined as the largest integer less than the original three-digit 1950 industry and occupational classifications divided by 100). $\gtrless$ 40 denote indicators for being above or below 40 years old at the time of enumeration. HS is an indicator for having 12 years of schooling. Standard errors clustered n state-year of birth. *** p<0.01, ** p<0.05, * p<0.1.

Table 5. Comparison to point estimators

	Inverse probability weighting				
	All years	1940	1950	1960	1970
Black	0.433***	0.494***	0.518***	0.456***	0.368***
	(0.00619)	(0.0141)	(0.0205)	(0.0105)	(0.0102)
N	87,413	21,259	8,037	28,612	29,505
White	0.183***	0.202***	0.229***	0.207***	0.140***
	(0.00362)	(0.00918)	(0.0133)	(0.00592)	(0.00569)
N	87,413	21,259	8,037	28,612	29,505

	Switching regression				
	All years	1940	1950	1960	1970
Black	0.918***	0.683***	1.020***	-0.883***	-0.837***
	(0.0296)	(0.0740)	(0.0913)	(0.0189)	(0.0192)
Observations	87,413	21,259	8,037	28,612	29,505
White	0.364***	0.413***	-1.321***	-1.102***	0.239***
	(0.0203)	(0.0483)	(0.0176)	(0.00866)	(0.0370)
Observations	297,405	61,939	24,442	97,094	113,930

Notes—Dependent variable is the log annual wage. Inverse probability weighting estimates based on race-specific propensity scores estimated using a logit model of Northward migration as a function of membership in five-year age groups and education groups (fewer than six, between 6 and 11, 12, or greater than 12 years of schooling), and for the pooled estimates, year effects. Reported ATTs calculated by regressing the log wage on interactions between age and education indicators and an indicator for migrating, weighting untreated observations by the ratio of their propensities for being treated and untreated, then calculating the implied difference in wages between the North and South, averaged according to the distribution of covariates among migrants. Switching regression estimates based on a model in which both the log wage and net benefit from migration are linear functions of age and education effects and jointly normally distributed errors. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.